Patterns Project

Overview

This five-part investigation tasks students with exploring sequences, both numeric and visual. Students will begin by exploring different types of growth with the goal of creating formula that describe the growth given the term number. Students will then focus on linear growth and begin to explore graphing a line using the standard form of an equation (y = mx+b).

1. Part I: Types of Growth: Students explore different types of number patterns and begin to categorize them by their type of growth.

2. Part II: Predicting Growth: Students look at pattern visualizations to describe growth and to create formulas that connect term number to value.

3. Part III: Graphing Growth: Students graph their findings and explore the different between linear and non-linear growth

4. Part IV: Linear Growth on the Graph: Students review how to graph a linear function using y-intercept and slope

5. Part V: Wrapping it Up!: Students practice their learning.

Learning Objectives

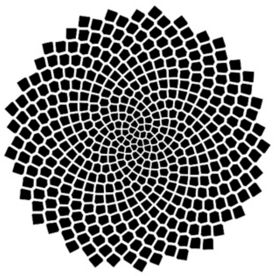
- CCSS.MATH.CONTENT.8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

- CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

- CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.

Teacher Notes

Name: ______________________________ Date: _______________



Part I: Types of Growth

Numbers can grow and shrink in infinite ways. In this exercise we are going to learn about different categories of growth and decay so that we can help better understand patterns!

Directions: Complete the number patterns below. Once you have filled in the next 4 numbers for each pattern, see if you can describe how each of the patterns is growing.

i	41, 37, 33, 29, ___ , ___ , ___ , ___	Can you describe the growth?
ii	12, 23, 34, 45, ___ , ___ , ___ , ___	Can you describe the growth?
iii	½, 2, 8, 32, ___ , ___ , ___ , ___	Can you describe the growth?
iv	100, 90, 190, 180, 280 , ___ , ___ , ___	Can you describe the growth?
v	1, 3, 7, 15, 31 , ___ , ___ , ___	Can you describe the growth?
vi	½ , 2, 3½, 5, ___ , ___ , ___ , ___	Can you describe the growth?

Reflection: Looking back at the patterns above, can you categorize them by their type of growth? What do they have in common? How are they different? See if you can categorize the patterns into different groups.

Your turn! Now that you have determined different types of growth, can you create 3 patterns of your own that fit some or all of these groups? This will depend on how you categorized your growth above.

vii	___ , ___ , ___ , ___ , ___ , …	What category would you put this pattern in?
viii	___ , ___ , ___ , ___ , ___ , …	What category would you put this pattern in?
ix	___ , ___ , ___ , ___ , ___ , …	What category would you put this pattern in?

Name: ____________________________ Date: ______________

Part II: Predicting Growth

The following visuals all represent patterns of growth. How can we describe their growth? Can we predict what the 5th, 10th, or 100th term will look like?

Directions: By observing the growth of each visual below, sketch the next graphic and respond to the prompts provided.

Pattern A

What will the 5th term look like?

n = 1 n = 2 n = 3 n = 4 n = 5

Fill in the table below using the visuals above. "n" refers to the term number. Can you come up with a formula that lets you predict the number of stars that the nth term will have?

n (term)	1	2	3	4	5	10	n (formula)
stars	1	3					

** turn the page for a hint on how to find the formula for the nth term!

Hint: There are two ways to describe pattern A: **recursively** or **relatively**. We should try and describe it both ways but our formula for the stars in the nth term should be a relative formula. See definitions below:

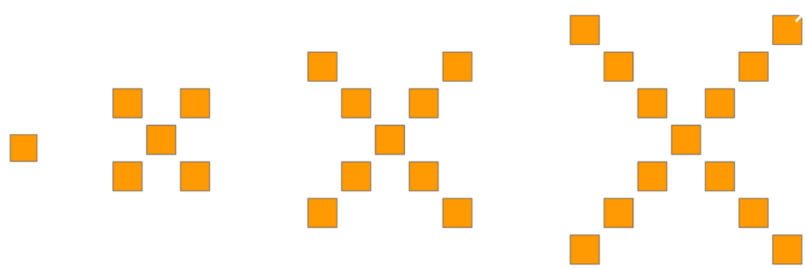
Recursive: a recursive formula describes the growth according to the term before it. For instance, if we said that the pattern grows by 2 stars each time this would be recursive because it depends on the previous term.

Relative: a relative formula describes the growth according to the **term number** (n) and does not require that you know how many items were in the previous term. For instance, the relative formula for pattern A (stars) is **2 x (term number) – 1** or **2n-1**

Pattern B

What will the 5th term look like?

n = 1 n = 2 n = 3 n = 4 n = 5

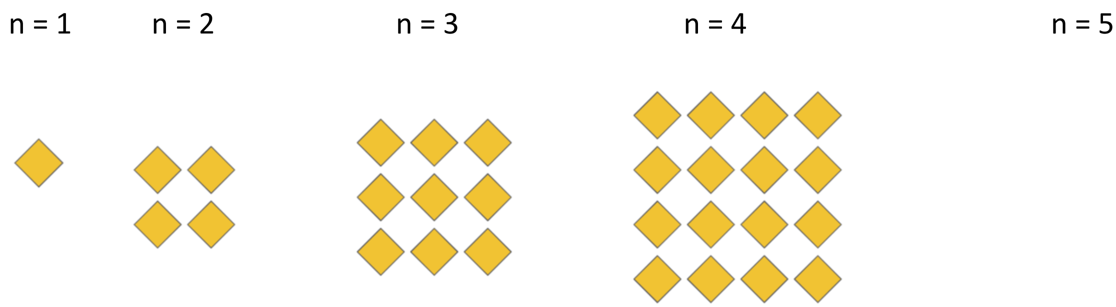


Fill in the table below using the visuals above. "n" refers to the term number. Can you come up with a formula that lets you predict the number of vertices and triangles that the nth term will have?

n (term)	1	2	3	4	5	10	n (formula)
squares							

Pattern C

What will the 5th term look like?



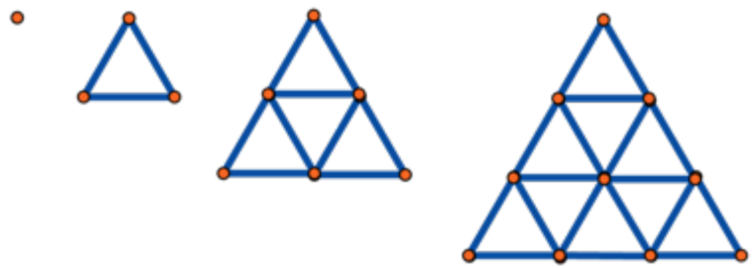
Fill in the table below using the visuals above. "n" refers to the term number. Can you come up with a formula that lets you predict the number of vertices, toothpicks and triangles that the nth term will have?

n (term)	1	2	3	4	5	10	n (formula)
diamonds							

Pattern D

What will the 5th term look like?

n = 1 n = 2 n = 3 n = 4 n = 5



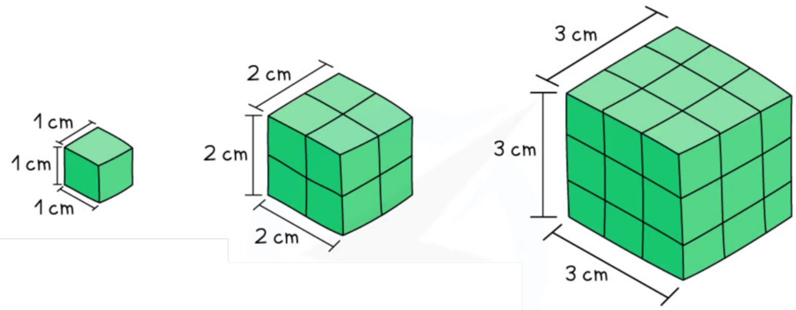
Fill in the table below using the visuals above. "n" refers to the term number. Can you come up with a formula that lets you predict the number of triangles that each term will have?

n (term)	1	2	3	4	5	10	n (formula)
triangles							

Pattern E :

What will the 4th term look like?

n = 1 n = 2 n = 3 n = 4



Fill in the table below using the visuals above. "n" refers to the term number. Can you come up with a formula that lets you predict the surface area and volume of the term?

n (term)	1	2	3	4	5	10	n (formula)
volume							
surface area							

Pattern F : Create your own visual pattern

Draw the first 4 terms of your pattern :

n = 1 n = 2 n = 3 n = 4

Fill in the table below using the visuals above. "n" refers to the term number. Can you come up with a formula that lets you predict some number of objects in your pattern? How many will the nth term have? Come up with as many objects as you'd like!

n (term)	1	2	3	4	5	10	n (formula)

Name: ___________________________ Date: _____________

Part III: Graphing Growth

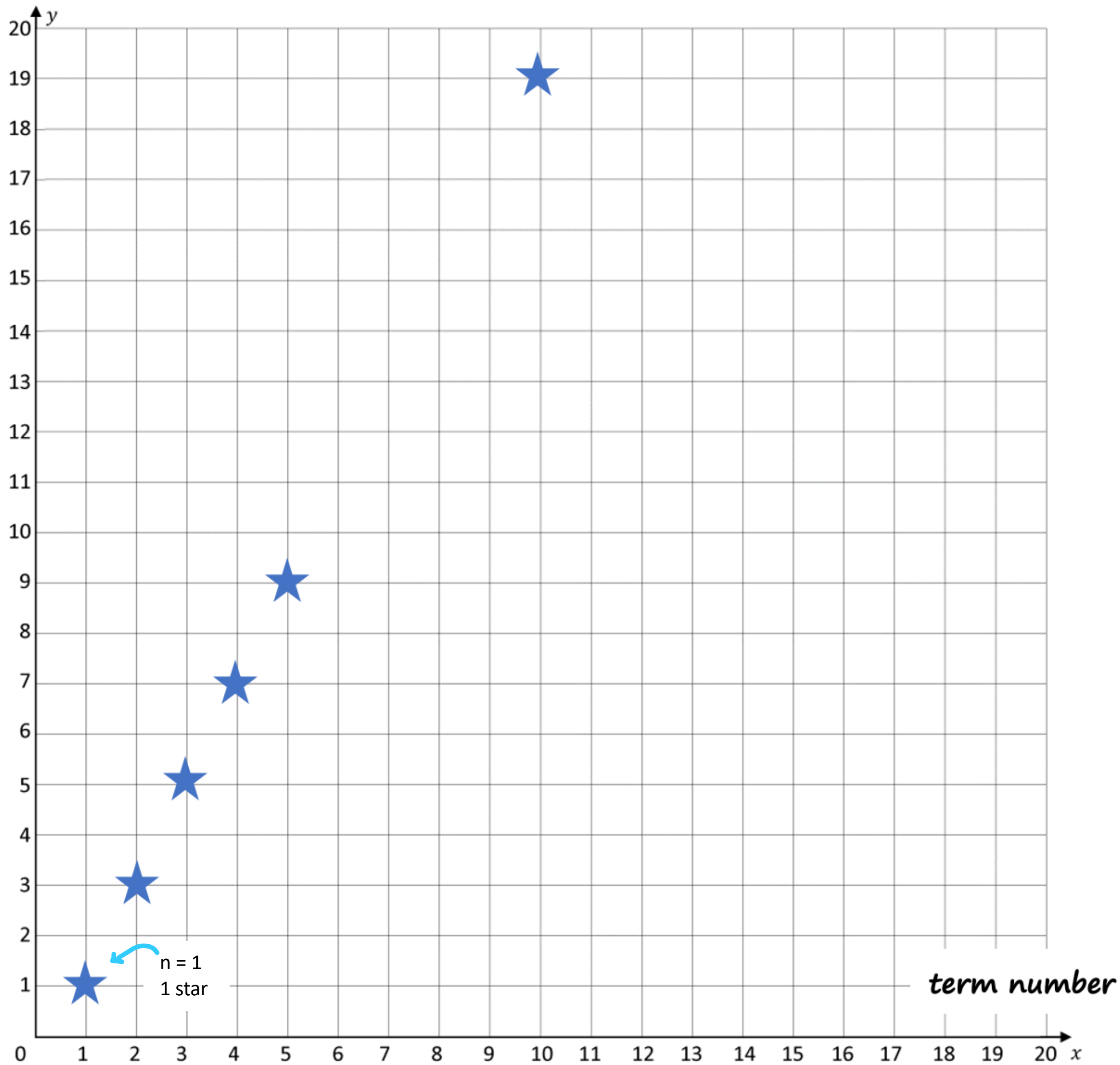
Growth can be described and visualized in all sorts of ways! One way that we can visualize growth is by graphing it. When we graph growth, we start by making sure we know what each axis of our graph represents. In this case, our axes should be labeled like this:

Where x-axis (our horizontal line) will keep track of our term number and the y-axis (the vertical line) will tell us what our item number or value is. This will change depending on the patterns. For instance, in pattern A we were keeping track of the **number of stars** we saw but in pattern E we were keeping track of the **surface area** and **volume** we calculated.

x-axis: the horizontal axis is called the x-axis and keeps track of the independent variable which is the variable that helps determine the value of the y-axis value. *For instance, how many hamburgers you eat determines how hungry you are so the number of hamburgers you eat is independent and how hungry you are is dependent.*

y-axis: the vertical axis is called the y-axis and keeps track of the dependent variable whose value will change depending on the x-axis.

Directions: On the following pages we are going to graph the growth that we observed in the patterns from part 2. Feel free to grab some colored pencils if you'd like to style out these graphs!

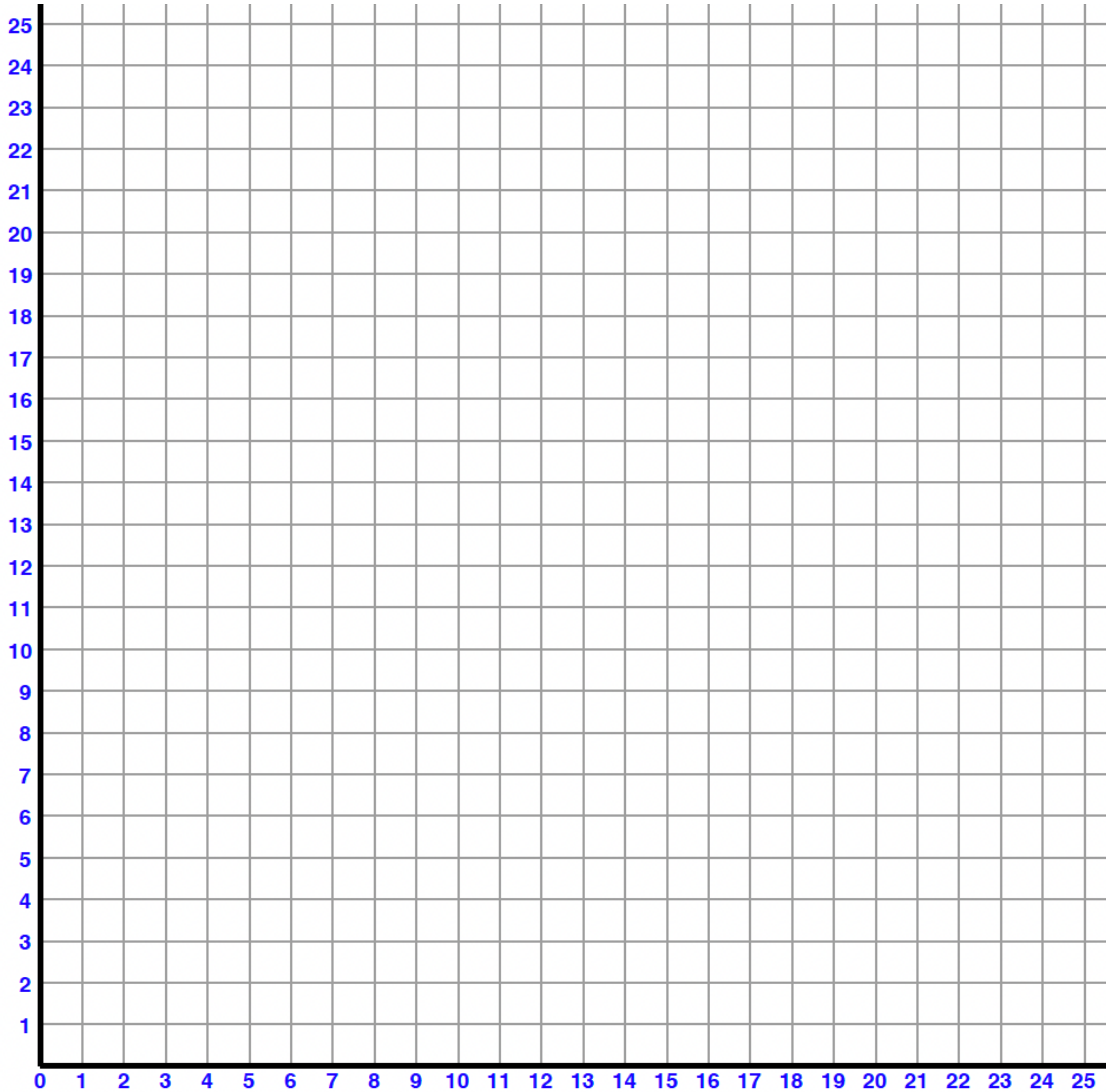


Directions: The graph of pattern A is drawn above. In order to graph patterns we need to look back at our tables from part II. If the first term (n = 1) had 1 star, then we graph the point (1,1) etc.

= Pattern A (term vs. # of stars)

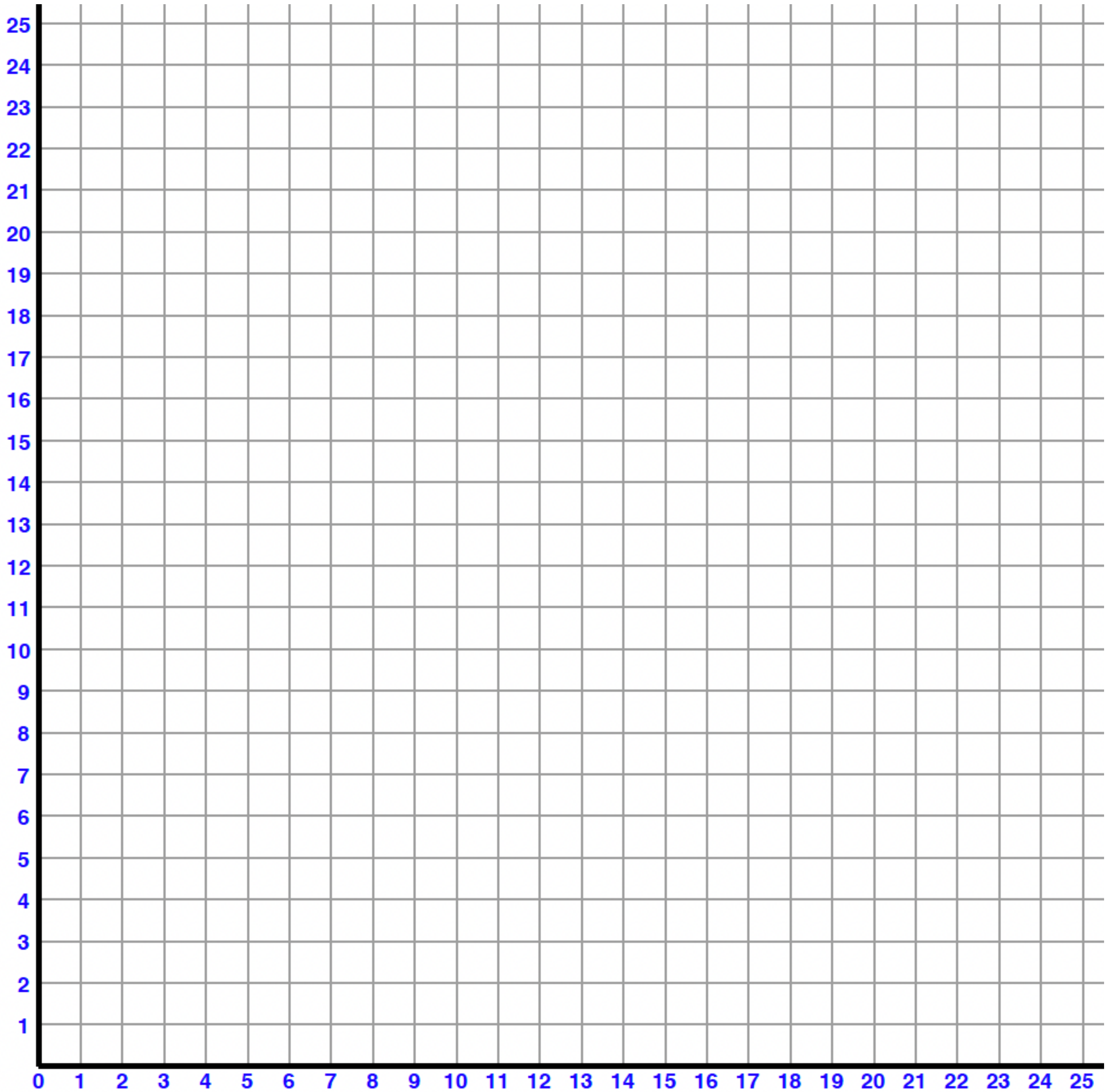
= Pattern B (term vs. # of squares)

Reflection: How many stars/blocks would the 8th term have? How do you know?



☐ = Pattern C (term vs. # of diamonds) ☐ = Pattern D (term vs. # of triangles)

Reflection: How does the growth of these patterns compare to the growth of patterns A and B?



= Pattern E (term vs. surface area)

= Pattern E (term vs. volume)

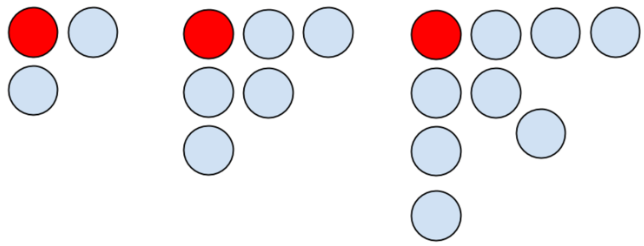
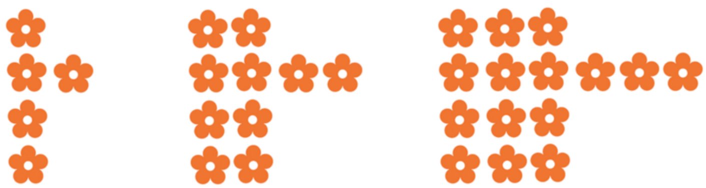
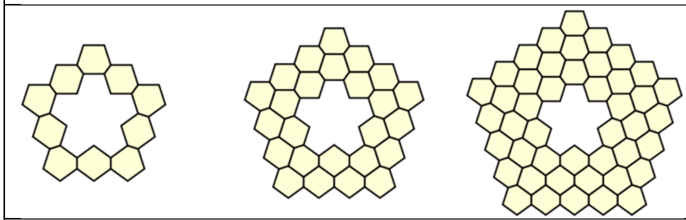
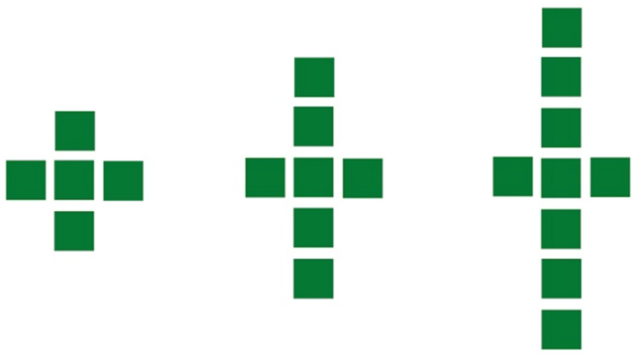
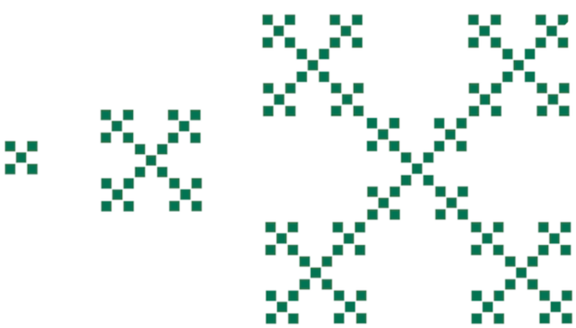
Reflection: Using just the graph can you easily predict the 6^{th} or 8^{th} term? Why or why not?

Part III continued: Graphing Reflection

1. Patterns A and B, as graphed above, are called ***linear*** because they create a ***line***. What is special about the way patterns A and B are growing that allows them to create a line when patterns C, D, and E do not?

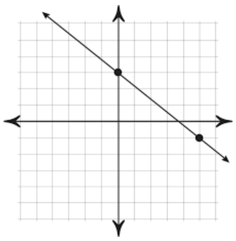
2. Now look back at patterns A & B in part II. How do you see the linear growth that you described in question 1 show up in the visual patterns?

3. Sum it up! Which of the following visual patterns would also have a linear growth?

	Does this pattern have linear growth? Explain!
	Does this pattern have linear growth? Explain!
	Does this pattern have linear growth? Explain!
	Does this pattern have linear growth? Explain!
	Does this pattern have linear growth? Explain!

Linear growth = ***A quantity grows linearly if it grows by a constant amount for each unit of time***.

Name: ______________________________ Date: _______________



Part IV: Linear Growth on the Graph

Now that we've learned about what linear growth looks like in a visual pattern we are going to look at how we can use equations to graph linear growth.

Directions:

1. Complete the following number patterns (like you did in part 1!)

i	0, 3, 6, 9, ___ , ___ , ___ , ___
ii	4, 7, 10, 13, ___ , ___ , ___ , ___
iii	1, 4, 7, 10, ___ , ___ , ___ , ___

2. What do these patterns have in common? How are they different?

3. Another way you will see these patterns written is in what is called a t-table. Convert the patterns you completed above into the t-tables below:

n	
1	
2	
3	
4	
5	
6	
7	
8	

n	
1	
2	
3	
4	
5	
6	
7	
8	

n	
1	
2	
3	
4	
5	
6	
7	
8	

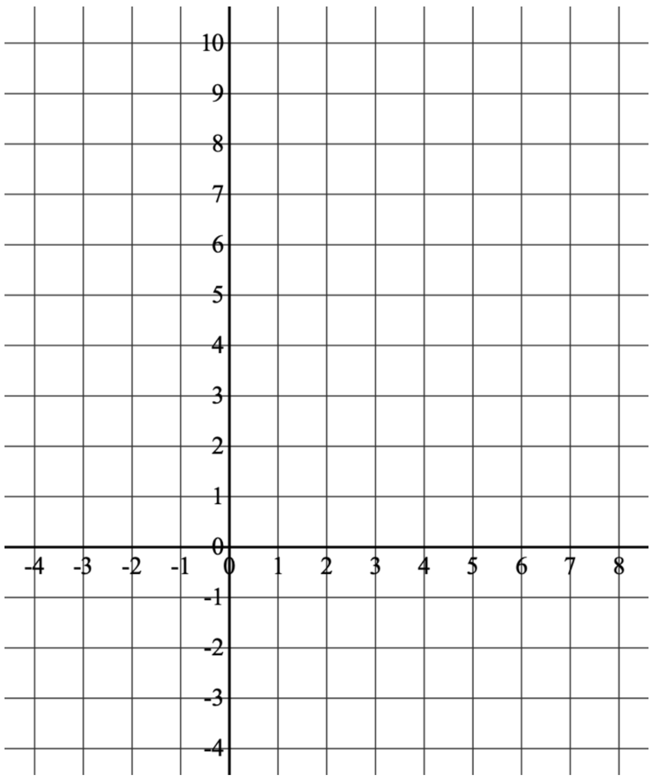
4. Now another change we are going to make with these t-tables is that we are actually going to ditch the term "n" which represented our term number, and call it "x". This will make it easier to transfer our data points onto a coordinate grid, which are more usually comparing the variables "x" and "y":

x	*y*		*x*	*y*		*x*	*y*
0			0			0	
1	0		1	4		1	1
2	3		2	7		2	4
3	6		3	10		3	7
4	9		4	13		4	10
5			5			5	
6			6			6	
7			7			7	
x			*x*			*x*	

5. Perhaps you've noticed that there is also a "0" now in the x-column. Can we predict what our patterns would be when x=0? Fill in the x=0 row above. How did you do it?

6. Last but not least! Can we determine what the formula for the x^{th} term would be? Fill in the formula on the bottom row of each table!

7. Now we graph!



= pattern a

= pattern b

= pattern c

8. What do you notice about the 3 patterns? How are they the same? How are they different? How does this relate to the formula you created in the previous question?

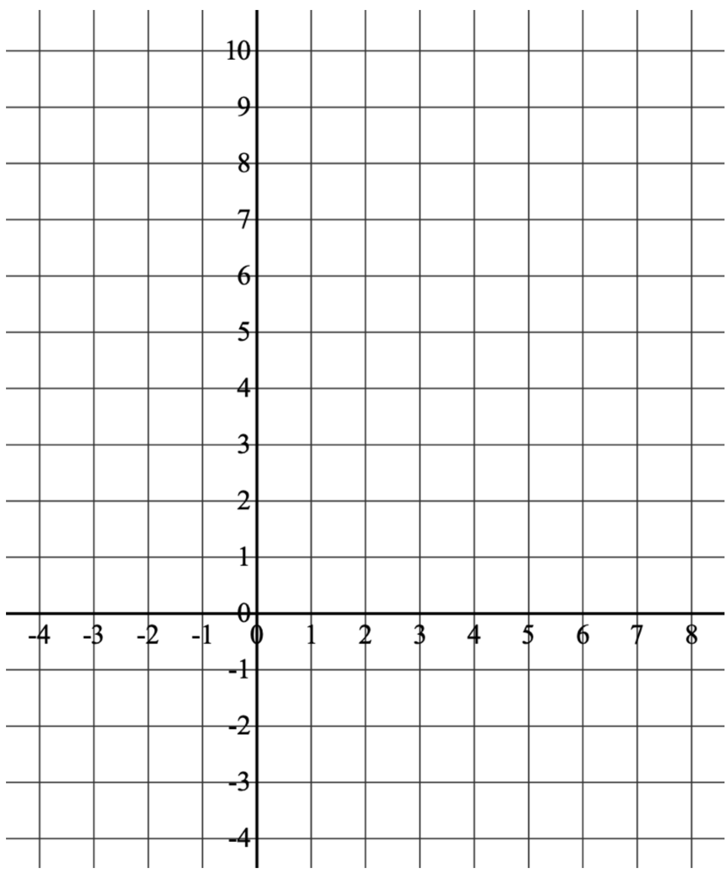
9. Let's try some different patterns… complete the following patterns:

x	*y*
0	2
1	5
2	8
3	11
4	
5	
x	

x	*y*
0	2
1	1
2	0
3	-1
4	
5	
x	

x	*y*
0	2
1	4
2	6
3	8
4	
5	
x	

Now we must graph!



☐ = pattern d

☐ = pattern e

☐ = pattern f

10. What do you notice about these 3 patterns? How are they similar? Different?

Name: ______________________________ Date: _______________

Part V: Wrapping it up!

A typical linear equation will be written in the following form:

$$y = mx + b$$

Where **x** in the term number, or the independent variable, **y** is the dependent variable, **b** is the y-intercept or where the line crosses the **y-axis** and **m** is the **slope** or the rate at which the equation changes as x grows by

1. See if these definitions make sense based on what you discovered in your own graphs above.

Now, can you use your own words to describe what m and b represent in the equation below?

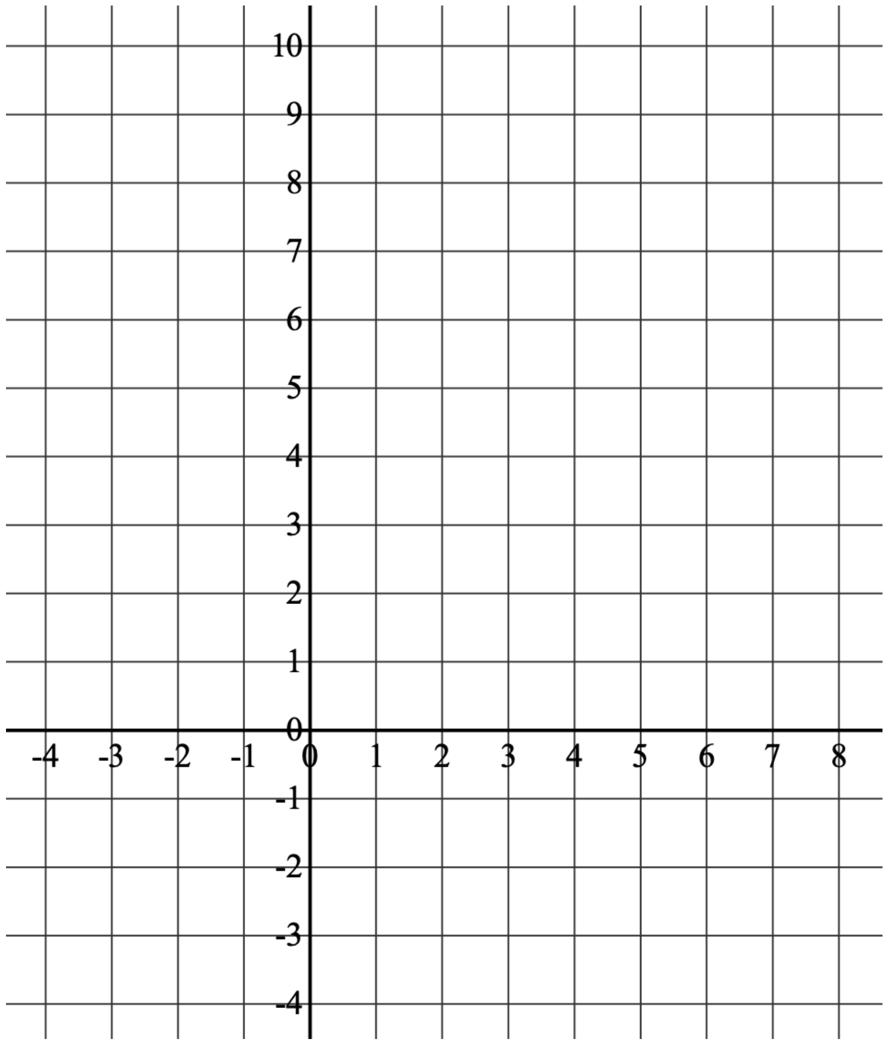
$$y = mx + b$$

We will test out our understanding by practicing our graphing on the following pages...

2. Graph the following equation, first by completing the t-table:

$$y = 2x + 2$$

x	*y*
0	
1	
2	
3	
4	
5	
6	
7	



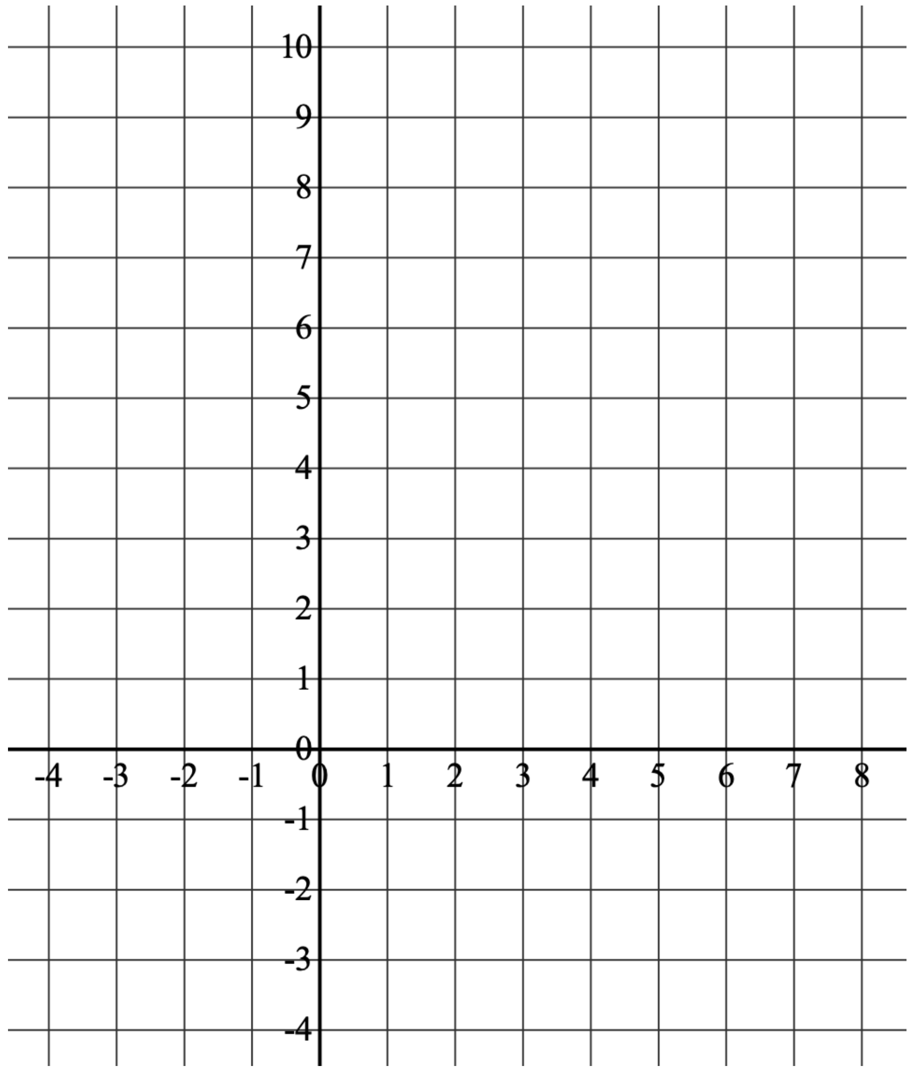
Where did the line cross the y-axis?

What was the slope, or rate of change, as x grew by 1? (how many units did the line go up every time it went over by 1?)

3. Graph the following equation, first by completing the t-table:

$$y = x + 2$$

x	*y*
0	
1	
2	
3	
4	
5	
6	
7	



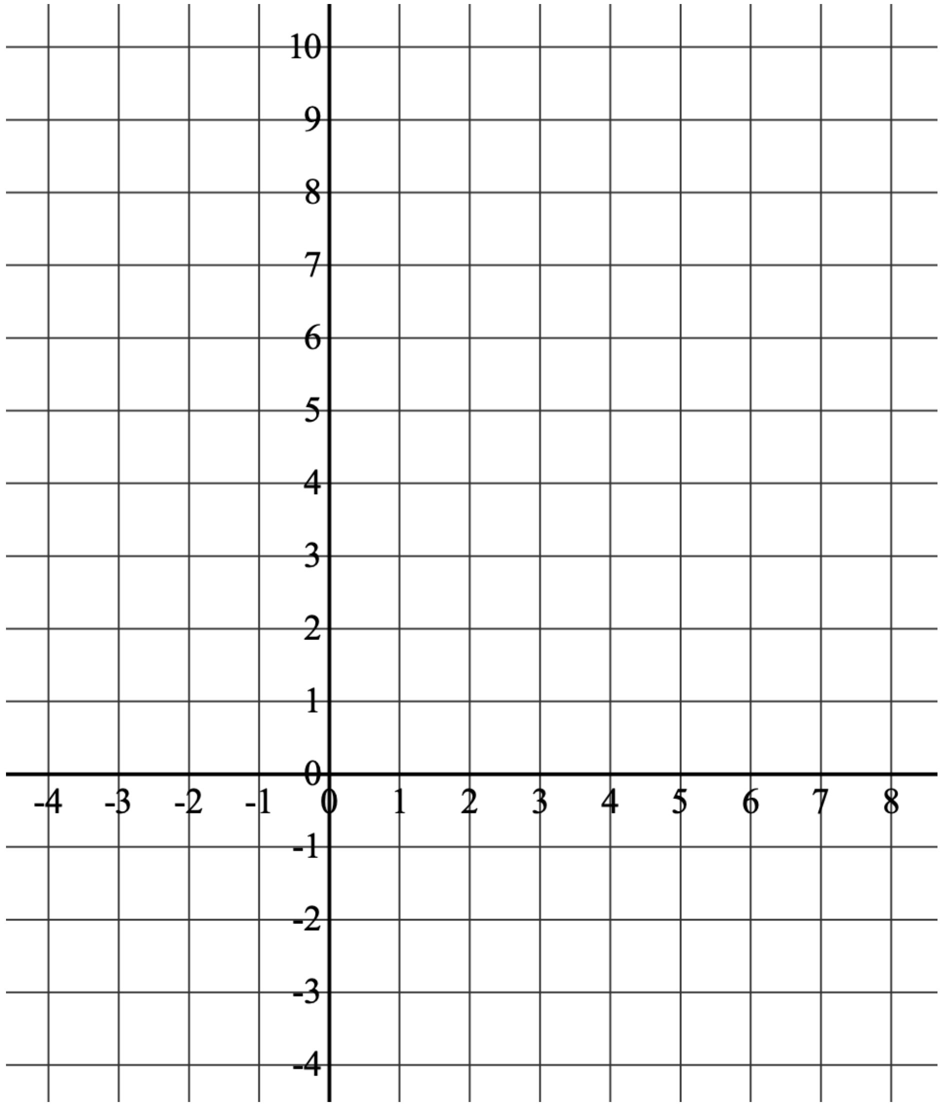
Where did the line cross the y-axis?

What was the slope, or rate of change, as x grew by 1? (how many units did the line go up every time it went over by 1?)

4. Graph the following equation, first by completing the t-table:

$$y = 4x - 1$$

x	y
0	
1	
2	
3	
4	
5	
6	
7	



Where did the line cross the y-axis?

What was the slope, or rate of change, as x grew by 1? (how many units did the line go up every time it went over by 1?)