Investigation: Factorization & Exponents

Warm Up: Let's start by proving that the following equation is true. What do both sides of the equation equal? Do they equal one another?

$$2 \text{ x } 3 \text{ x } 12 \text{ x } 5 = 6 \text{ x } 2 \text{ x } 6 \text{ x } 5$$

Now that we agree with the equation above (*we think that both sides are equal*) can you think of a way to prove this without actually multiplying all of the terms together?

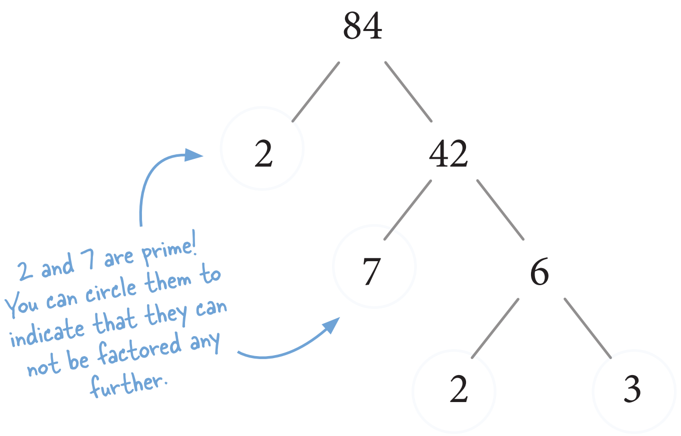
1. What we learned from our warm up was that there are many combinations of factors that multiply to the same product. Using that idea, can you think of 3 more ***unique*** ways in which you can multiply to 360? Remember that order doesn't matter when we multiply numbers together. For example, $5 \text{ x } 2$ is **<u>not</u>** unique from $2 \text{ x } 5$.

______________________________ $= 360$

______________________________ $= 360$

______________________________ $= 360$

2. Next, we recall that when we find the prime factorization of any **composite number** we factor our composite number into as many prime numbers as possible. For example, to find the prime factorization of 84, we do make can make the following factor tree:



Composite number:
A composite number is a number that has factors other than 1 and itself. It is the opposite of a prime number

prime-factorization of 84: 2 x 7 x 2 x 3 = 84

Your turn! Practice finding the prime-factorization of the following numbers:

a. 96

b. 144

c. 320

d. 890

3. One thing we notice about prime factorizations is that many of them have repeated factors. For instance, in our solution to **2a** we should have had:

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

Recall that order doesn't matter for factorization! If your answer had the same factors, in a different order, you did it correctly!!

We can rewrite this using our knowledge of exponents as:

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

$$2^5 \times 3 = 96$$

Can any of the other solutions to **problem 2** be rewritten with exponents?

144 = ____________________________________

320 = ____________________________________

890 = ____________________________________

4. We also notice that some prime factorizations can be split up into two identical groups. For instance, **3b** can be written as:

$$\underbrace{(2 \times 2 \times 3)}_{\text{group 1}} \cdot \underbrace{(2 \times 2 \times 3)}_{\text{group 2}} = 144$$

Do we see that each group has the same combination of prime factors? What do you think this might tell us about our product? **Take a guess!**

____

4. continued...

Turns out that if we can split up our prime factors into two identical groups we know that we have found a perfect square!

Can you use prime factorization to determine if any of the following are perfect squares?

a. 200

b. 256

c. 900

d. 64

Extra Challenge:
Do you notice anything special about the prime factors of 64? Could you split them into more than just two equal groups? What does this suggest about the number 64...

5. Putting it all together...

Now let's combine all of our findings and solve some more challenging problems. For the following problems, fill in the blank with the appropriate value to make the equation true:

a. $3 \times 5 \times 3 \times 5 = \square^2 \times \square^2 = \square^2$

b. $648 = \square^3 \times \square^2$

c. $\square^2 = 3^4 \times 4^3$

Investigation: Factorization & Exponents

Warm Up: Let's start by proving that the following equation is true. What do both sides of the equation equal? Do they equal one another?

$$2 \times 3 \times 12 \times 5 = 6 \times 2 \times 6 \times 5$$

Now that we agree with the equation above (*we think that both sides are equal*) can you think of a way to prove this without actually multiplying all of the terms together?

Answers may vary.

One way we can think about proving that this equation is true is by showing that the factors on the left can we re-arranged or combined to create the same factors on the right.

$$2 \times 3 \times 12 \times 5 = 6 \times 2 \times 6 \times 5$$

$= 6 \quad = 2 \times 6$

$$6 \times 2 \times 6 \times 5 = 6 \times 2 \times 6 \times 5$$

1. What we learned from our warm up was that there are many combinations of factors that multiply to the same product. Using that idea, can you think of 3 more ***unique*** ways in which you can multiply to 360? Remember that order doesn't matter when we multiply numbers together. For example, 5×2 is **not** unique from 2×5.

Answers may vary.

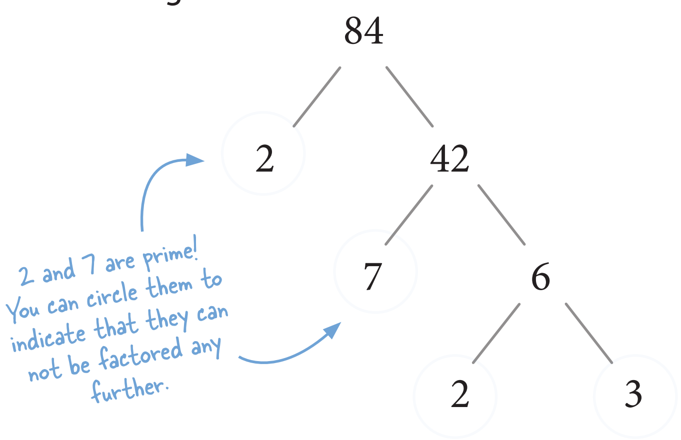
If we continue to work with our factorizations from the warm-up, we can find lots of different ways to combine and multiply the factors - just as long as we keep track of which factors we've combined and which ones remain.

$2 \times 3 \times 3 \times 4 \times 5 = 360$

$2 \times 3 \times 3 \times 2 \times 2 \times 5 = 360$

$18 \times 2 \times 10 = 360$

2. Next, we recall that when we find the prime factorization of any **composite number** we factor our composite number into as many prime numbers as possible. For example, to find the prime factorization of 84, we do make can make the following factor tree:



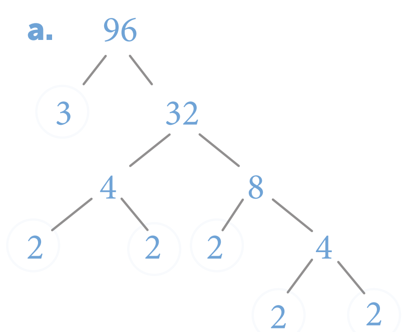
> **Composite number:**
> A composite number is a number that has factors other than 1 and itself. It is the opposite of a prime number

prime-factorization of 84: 2 x 7 x 2 x 3 = 84

Your turn! Practice finding the prime-factorization of the following numbers:

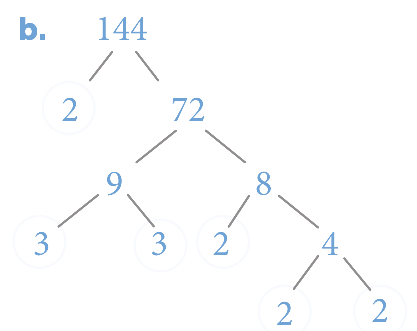
Answers may vary. *To completely prime-factorize a number we use a factor tree. It doesn't matter what factor pair you choose first when you make your first branches but make sure you keep going until each number that remains is a prime number!*

a.



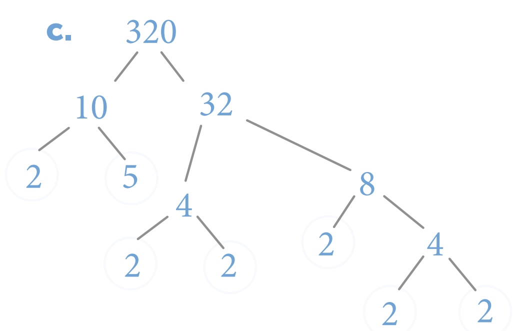
3 x 2 x 2 x 2 x 2 x 2 = 96

b.



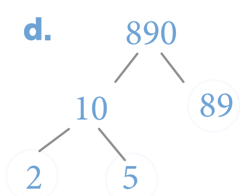
2 x 3 x 3 x 2 x 2 x 2 = 144

c.



2 x 5 x 2 x 2 x 2 x 2 x 2 = 320

d.



2 x 5 x 89 = 890

3. One thing we notice about prime factorizations is that many of them have repeated factors. For instance, in our solution to **2a** we should have had:

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

Recall that order doesn't matter for factorization! if your answer had the same factors, in a different order, you did it correctly!!

We can rewrite this using our knowledge of exponents as:

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

$$2^5 \times 3 = 96$$

Can any of the other solutions to **problem 2** be rewritten with exponents?

Step 1. *Re-write the prime-factorization by grouping factors together, typically least to greatest:*

Step 2. *Rewrite any repeat factors using exponent notation:*

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

$$320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$$

$$890 = 2 \times 5 \times 89$$

No repeat factors? No need to rewrite!

4. We also notice that some prime factorizations can be split up into two identical groups. For instance, **3b** can be written as:

$$\underbrace{(2 \times 2 \times 3)}_{\text{group 1}} \cdot \underbrace{(2 \times 2 \times 3)}_{\text{group 2}} = 144$$

Do we see that each group has the same combination of prime factors? What do you think this might tell us about our product? **Take a guess!**

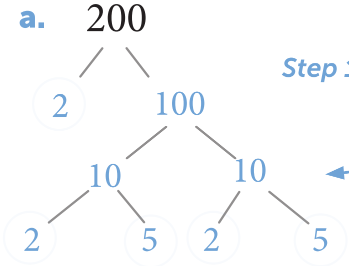
If we can split up our factors into two identical groups (like in our example above) this means that our number must be a perfect square because you can create 2 identical factor groups. In the example above we see that $(2 \times 2 \times 3) \cdot (2 \times 2 \times 3) = (12) \cdot (12) = 144$*, or* $12^2 = 144$*.*

4. continued...

Turns out that if we can split up our prime factors into two identical groups we know that we have found a perfect square!

Can you use prime factorization to determine if any of the following are perfect squares?

a. 200



Step 1. *Prime-factorize*

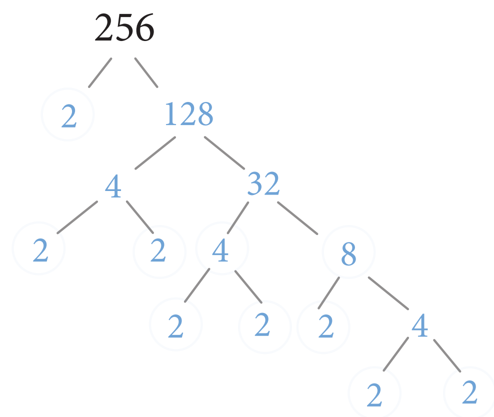
$2 \times 2 \times 5 \times 2 \times 5 = 200$

$(2 \times 5) \cdot (2 \times 5) \times 2 = 200$

200 is not a perfect square!

Step 2. *If possible, create two identical groups of factors. If this is possible, you have found a perfect square. If not, your number is not a perfect square.*

b. 256



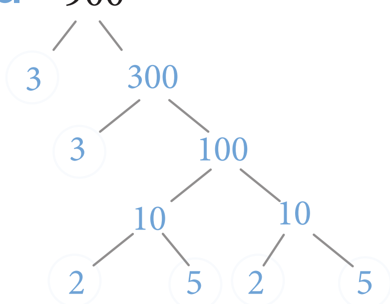
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$(2 \times 2 \times 2 \times 2) \cdot (2 \times 2 \times 2 \times 2) = 256$

$(16) \cdot (16) = 16^2 = 256$

256 is a perfect square!

c. 900



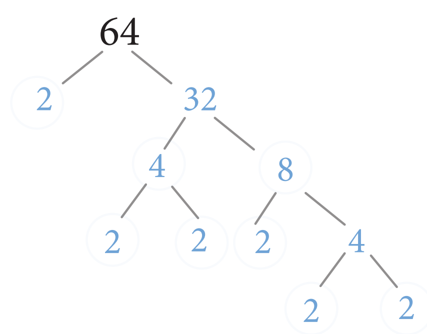
$2 \times 2 \times 3 \times 3 \times 5 \times 5 = 900$

$(2 \times 3 \times 5) \cdot (2 \times 3 \times 5) = 900$

$(30) \cdot (30) = 30^2 = 900$

900 is a perfect square!

d. 64



$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

$(2 \times 2 \times 2) \cdot (2 \times 2 \times 2) = 256$

$(8) \cdot (8) = 8^2 = 64$

64 is a perfect square!

Extra Challenge:
Do you notice anything special about the prime factors of 64? Could you split them into more than just two equal groups? What does this suggest about the number 64...

Not only can we split up the factors of 64 into two groups, we can split up the factors of 64 into three identical groups, proving that 64 is also a perfect cube!

$(2 \times 2) \cdot (2 \times 2) \cdot (2 \times 2) = 64$

$(4) \cdot (4) \cdot (4) = 4^3 = 64$

5. Putting it all together...

Now let's combine all of our findings and solve some more challenging problems. For the following problems, fill in the blank with the with the appropriate value to make the equation true:

a. $3 \times 5 \times 3 \times 5 = \boxed{3}^2 \times \boxed{5}^2 = \boxed{15}^2$

$3 \times 3 \times 5 \times 5$	***Step 1.*** *By grouping our factors together by value we see that we have two 3s and two 5s.*
$3^2 \times 5^2$	***Step 2.*** *Re-write these in exponential notation.*
$(3 \times 5) \cdot (3 \times 5) = (15) \cdot (15)$	***Step 3.*** *We also know this number is a perfect square because we can group the factors into equal groups.*

b. $648 = \boxed{9}^2 \times \boxed{2}^3$

$3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$	***Step 1.*** *Find the prime-factorization of 648*
$3^4 \times 2^3$	***Step 2.*** *Re-write the factors in exponential notation.*
$(3 \times 3) \cdot (3 \times 3) \cdot 2^3$	***Step 3.*** *We can see that our cubed factor must be 2, so we must then make our squared factor with our* $3 \times 3 \times 3 \times 3$. *Split up* 3^4 *into* $3^2 \times 3^2$.
$9^2 \times 2^3$	***Step 4.*** *Re-write as* $3 \times 3 \times 3 \times 3$ *as* 9×9 *or* 9^2

c. $\boxed{108}^2 = 3^4 \times 4^3$

$3^4 \times 4^3$	***Step 1.*** *Because we know that 4 is not a prime, we can continue to break down our factors on the left side.*
$3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \quad \times 2 \times 2 \times 2$	***Step 2.*** *Break up each exponent into its prime factors*
$(3 \times 3 \times 2 \times 2 \times 2) \cdot (3 \times 3 \times 2 \times 2 \times 2)$	***Step 3.*** *Create two equal groups of prime-factors*
$(108) \cdot (108)$	***Step 4.*** *Multiple the factors of each group together*
108^2	***Step 5.*** *Re-write in exponential notation*